51. **Answer (B):** By Euler's formula the expression $e^{i2\pi j\alpha} = 1$ holds if $2\pi j\alpha$ is an integer multiple of 2π. Hence $j\alpha$ must be rational and this is only true when $j = 0$. Hence there is only one solution.

52. **Answer (C):** To determine a champion, $n-1$ teams must be eliminated. This requires $n-1$ games.

53. **Answer (D):** Setting up a Venn diagram, the number of students taking some AP class is 62. Therefore 38 students are not taking a class in these three areas.

54. **Answer (C):** Since the given ordered pairs for $r(x)$ include two changes of sign of the slope, $r(x)$ is uniquely determined and $p(x)$ may be expressed as $p(x) = r(x) + C(x-1)(x-2)(x-3)(x-4)$, where C is a constant. Set x to 5, then $-36 = p(5) = r(5) + 24C$, so $C = -\frac{3}{2}$. This is the value of the coefficient of x^4 in $p(x)$.

OR

Since the value of x provided are ascending adjacent integers, take four rounds of successive differences in the values for $p(x)$ and divide by 4!:

4		3		6		7		-36
	-1		3		1		-43	
		4		-2		-44		
			-6		-42			
				-36				

$\frac{-36}{24} = -\frac{3}{2}$. This solution method requires knowledge of calculus, but it seems reasonable to assume that it takes longer to apply than the solution method above.

Note: The functions are:

$$r(x) = -x^3 + 8x^2 - 18x + 15 \quad \text{and}$$
$$p(x) = -\frac{3}{2}x^4 + 14x^3 - \frac{89}{2}x^2 + 57x - 21$$

55. **Answer (D):** Combining logarithms and exponentiating results in

$$\begin{aligned}\log_{10}(8\sin x) + \log_{10}(5\cos x) &= 1,\\ \log_{10}(40\sin x\cos x) &= 1,\\ 40\sin x\cos x &= 10.\end{aligned}$$

Applying the double angle for sine and simplifying gives,

$$20\sin 2x = 10,$$
$$\sin 2x = \frac{1}{2}.$$

Because x is between $\frac{\pi}{4}$ and $\frac{\pi}{2}$, we have that $2x$ is between $\frac{\pi}{2}$ and π. Therefore $2x = \frac{5\pi}{6}$, and $x = \frac{5\pi}{12}$.

56. Answer: 9. In order for $\frac{3^n}{n^3}$ to be an integer, n must be a power of 3, say $n = 3^m$. Substituting $3^{3^m}/3^{3m} = 3^{3^m-3m} = 3^3$. Then $3^m - 3m = 3$. Testing, $m = 2$, so $n = 3^2 = 9$.

58. **Answer (C):** Let us depict all pairs of x and y between 1 and 2011, inclusive, as two-dimensional points forming a square. The points for which $x = y$ form a diagonal of the square and there are 2011 such points. The points for which $|x - y| = 1$ form two straight lines immediately above and below that diagonal, each of which consists of 2010 points. The points for which $|x - y| = 2$ form two straight lines two steps above and below that diagonal, each of which consists of 2009 points. Therefore, the answer is $(2*2009+2*2010+2011)/2011^2 \approx 0.0025$.

59. **Answer (D):** If the first player draws a card labeled x and the second player draws a card labeled y the second player receives xy pennies. This occurs with probability $1/(25*25)$. The expected value of the game is then $\sum_{x=1}^{25}\sum_{y=1}^{25} xy \cdot 1/(25*25) = \frac{1}{25\cdot 25}\sum_{x=1}^{25} x\sum_{y=1}^{25} y = \frac{1}{25\cdot 25}\frac{25\cdot 26}{2}\frac{25\cdot 26}{2} = 13\cdot 13 = 169$.

60. **Answer (D):** Triangle ACD is a 3-4-5 right triangle so $\triangle ABE \sim \triangle ADC$ and $BD = \frac{3}{4}AB$. Because $\triangle ABE$ is half $\triangle ACD$, $\frac{1}{2}(AB)(BE) = \frac{1}{2}(\frac{1}{2})(CD)(AD) = \frac{1}{4}(12) = 3$. Therefore $\frac{1}{2}AB(\frac{3}{4}AB) = 3$, and $AB = 2\sqrt{2}$.

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61. **Answer (A):** Solving for x gives

$$x = \frac{2013}{1+2+3+\cdots+2012} = \frac{2013}{\frac{2012\cdot 2013}{2}} = \frac{1}{1006}$$

Therefore

$$2013x + 2014x + 2015x + \cdots + 4024x$$

$$
\begin{aligned}
&= x(1 + 2 + 3 + \cdots + 4024) - (x + 2x + 3x + \cdot + 2012x) \\
&= \frac{1}{1006} \cdot \frac{4024 \cdot 4025}{2} - 2013 \\
&= 2 \cdot 4025 - 2013 = 6037.
\end{aligned}
$$