

51. **Answer (B):** By Euler's formula the expression $e^{i2\pi j\alpha} = 1$ holds if $2\pi j\alpha$ is an integer multiple of 2π . Hence $j\alpha$ must be rational and this is only true when $j = 0$. Hence there is only one solution.
52. **Answer (C):** To determine a champion, $n - 1$ teams must be eliminated. This requires $n - 1$ games.
53. **Answer (D):** Setting up a Venn diagram, the number of students taking some AP class is 62. Therefore 38 students are not taking a class in these three areas.
54. **Answer (C):** Since the given ordered pairs for $r(x)$ include two changes of sign of the slope, $r(x)$ is uniquely determined and $p(x)$ may be expressed as $p(x) = r(x) + C(x - 1)(x - 2)(x - 3)(x - 4)$, where C is a constant. Set x to 5, then $-36 = p(5) = r(5) + 24C$, so $C = -\frac{3}{2}$. This is the value of the coefficient of x^4 in $p(x)$.

OR

Since the value of x provided are ascending adjacent integers, take four rounds of successive differences in the values for $p(x)$ and divide by $4!$:

$$\begin{array}{ccccccc}
 4 & & 3 & & 6 & & 7 & & -36 \\
 & -1 & & 3 & & 1 & & -43 & \\
 & & 4 & & -2 & & -44 & & \\
 & & & -6 & & -42 & & & \\
 & & & & -36 & & & &
 \end{array}$$

$\frac{-36}{24} = -\frac{3}{2}$. This solution method requires knowledge of calculus, but it seems reasonable to assume that it takes longer to apply than the solution method above.

Note: The functions are:

$$\begin{aligned}
 r(x) &= -x^3 + 8x^2 - 18x + 15 \quad \text{and} \\
 p(x) &= -\frac{3}{2}x^4 + 14x^3 - \frac{89}{2}x^2 + 57x - 21
 \end{aligned}$$

55. **Answer (D):** Combining logarithms and exponentiating results in

$$\begin{aligned}
 \log_{10}(8 \sin x) + \log_{10}(5 \cos x) &= 1, \\
 \log_{10}(40 \sin x \cos x) &= 1, \\
 40 \sin x \cos x &= 10.
 \end{aligned}$$

Applying the double angle for sine and simplifying gives,

$$\begin{aligned} 20 \sin 2x &= 10, \\ \sin 2x &= \frac{1}{2}. \end{aligned}$$

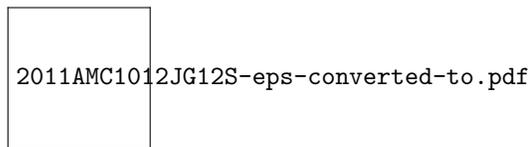
Because x is between $\frac{\pi}{4}$ and $\frac{\pi}{2}$, we have that $2x$ is between $\frac{\pi}{2}$ and π . Therefore $2x = \frac{5\pi}{6}$, and $x = \frac{5\pi}{12}$.

56. **Answer:** 9. In order for $\frac{3^n}{n^3}$ to be an integer, n must be a power of 3, say $n = 3^m$. Substituting $3^{3^m}/3^{3^m} = 3^{3^m-3m} = 3^3$. Then $3^m - 3m = 3$. Testing, $m = 2$, so $n = 3^2 = 9$.

58. **Answer (C):** Let us depict all pairs of x and y between 1 and 2011, inclusive, as two-dimensional points forming a square. The points for which $x = y$ form a diagonal of the square and there are 2011 such points. The points for which $|x - y| = 1$ form two straight lines immediately above and below that diagonal, each of which consists of 2010 points. The points for which $|x - y| = 2$ form two straight lines two steps above and below that diagonal, each of which consists of 2009 points. Therefore, the answer is $(2 \cdot 2009 + 2 \cdot 2010 + 2011)/2011^2 \approx 0.0025$.

59. **Answer (D):** If the first player draws a card labeled x and the second player draws a card labeled y the second player receives xy pennies. This occurs with probability $1/(25 \cdot 25)$. The expected value of the game is then $\sum_{x=1}^{25} \sum_{y=1}^{25} xy \cdot 1/(25 \cdot 25) = \frac{1}{25 \cdot 25} \sum_{x=1}^{25} x \sum_{y=1}^{25} y = \frac{1}{25 \cdot 25} \frac{25 \cdot 26}{2} \frac{25 \cdot 26}{2} = 13 \cdot 13 = 169$.

60. **Answer (D):** Triangle ACD is a 3-4-5 right triangle so $\triangle ABE \sim \triangle ADC$ and $BD = \frac{3}{4}AB$. Because $\triangle ABE$ is half $\triangle ACD$, $\frac{1}{2}(AB)(BE) = \frac{1}{2}(\frac{1}{2})(CD)(AD) = \frac{1}{4}(12) = 3$. Therefore $\frac{1}{2}AB(\frac{3}{4}AB) = 3$, and $AB = 2\sqrt{2}$.



61. **Answer (A):** Solving for x gives

$$x = \frac{2013}{1 + 2 + 3 + \cdots + 2012} = \frac{2013}{\frac{2012 \cdot 2013}{2}} = \frac{1}{1006}$$

Therefore

$$2013x + 2014x + 2015x + \cdots + 4024x$$

$$\begin{aligned} &= x(1 + 2 + 3 + \cdots + 4024) - (x + 2x + 3x + \cdots + 2012x) \\ &= \frac{1}{1006} \cdot \frac{4024 \cdot 4025}{2} - 2013 \\ &= 2 \cdot 4025 - 2013 = 6037. \end{aligned}$$